Bohr Model of the Atom (1914)

- Ground State: The atom has a lowest energy level, $E \neq 0$.
  - No more energy radiated (for now)
  - Bohr states the fact that matter does not collapse!
- Excited states: The atom can be in a set of higher energy configurations, with given energies, E_j (Energy levels).
  - This configurations are relatively stable
  - Discrete energy levels
- Transition between atomic states occur via the absorption or emission of a photon, $E_\gamma = hf$. The quantum of energy matches the energy difference between the levels.

$$E_\gamma = hf = \frac{hc}{\lambda} = E_a - E_b$$

➢ Atoms absorb and emit photons of particular $\lambda's$: Line spectrum

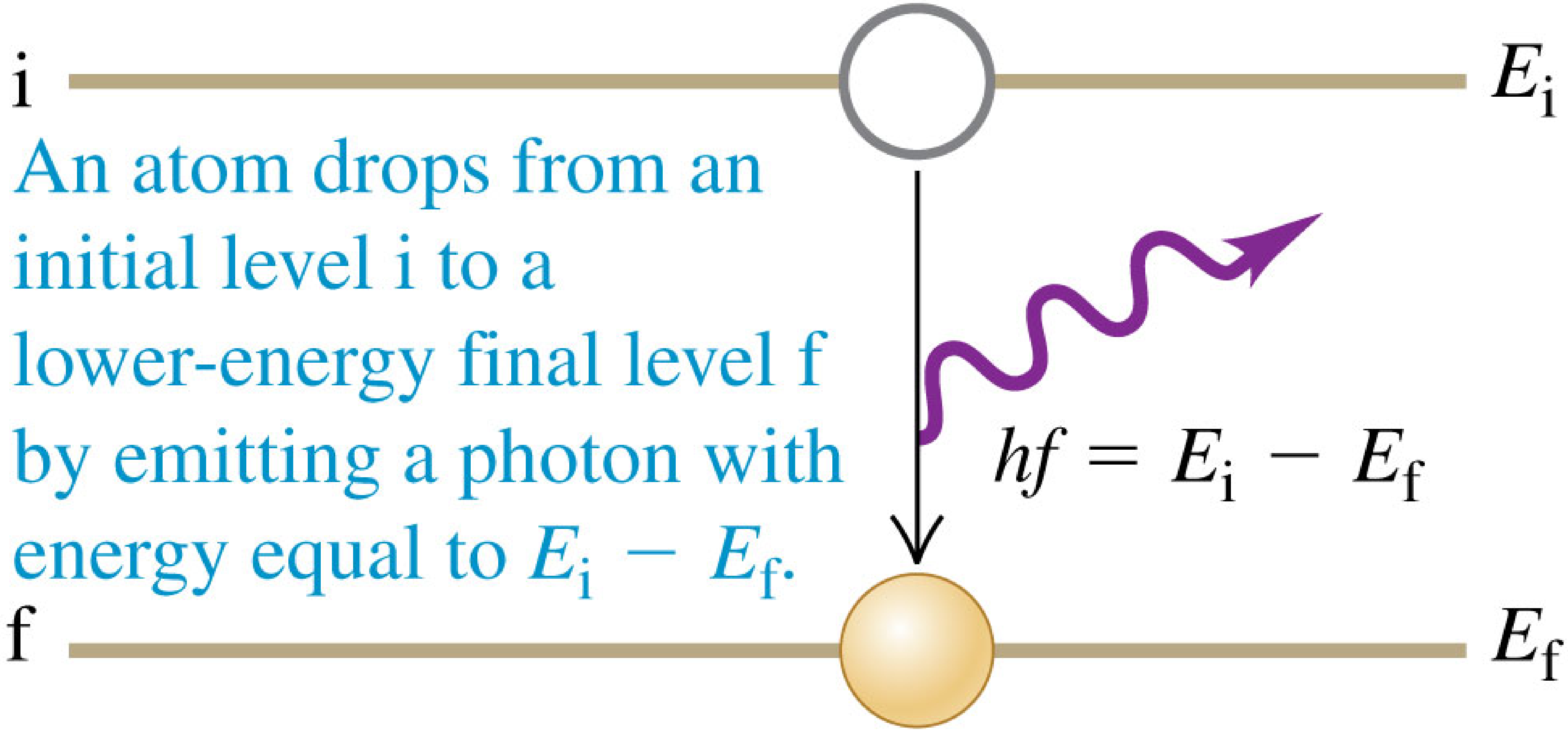
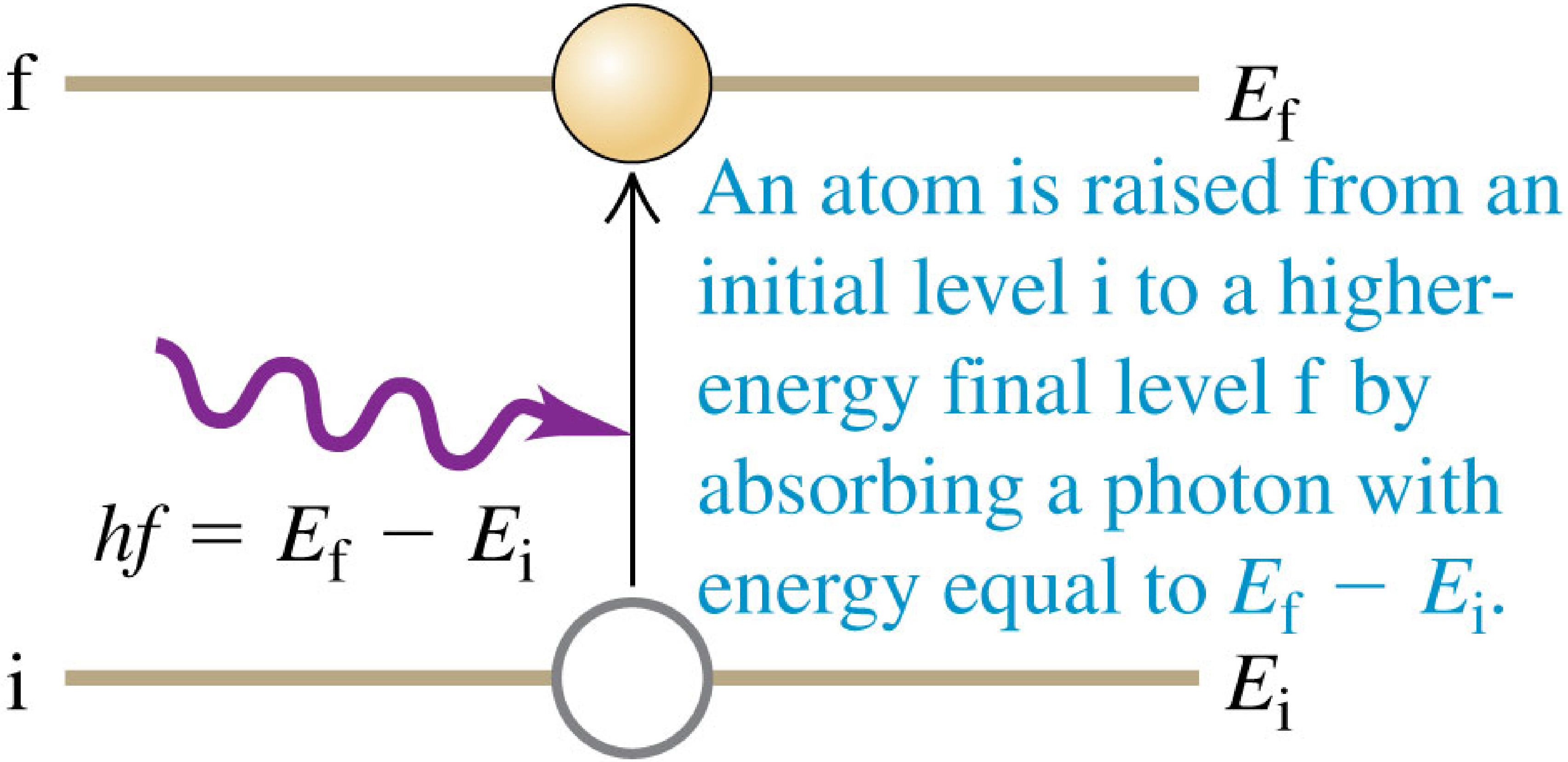
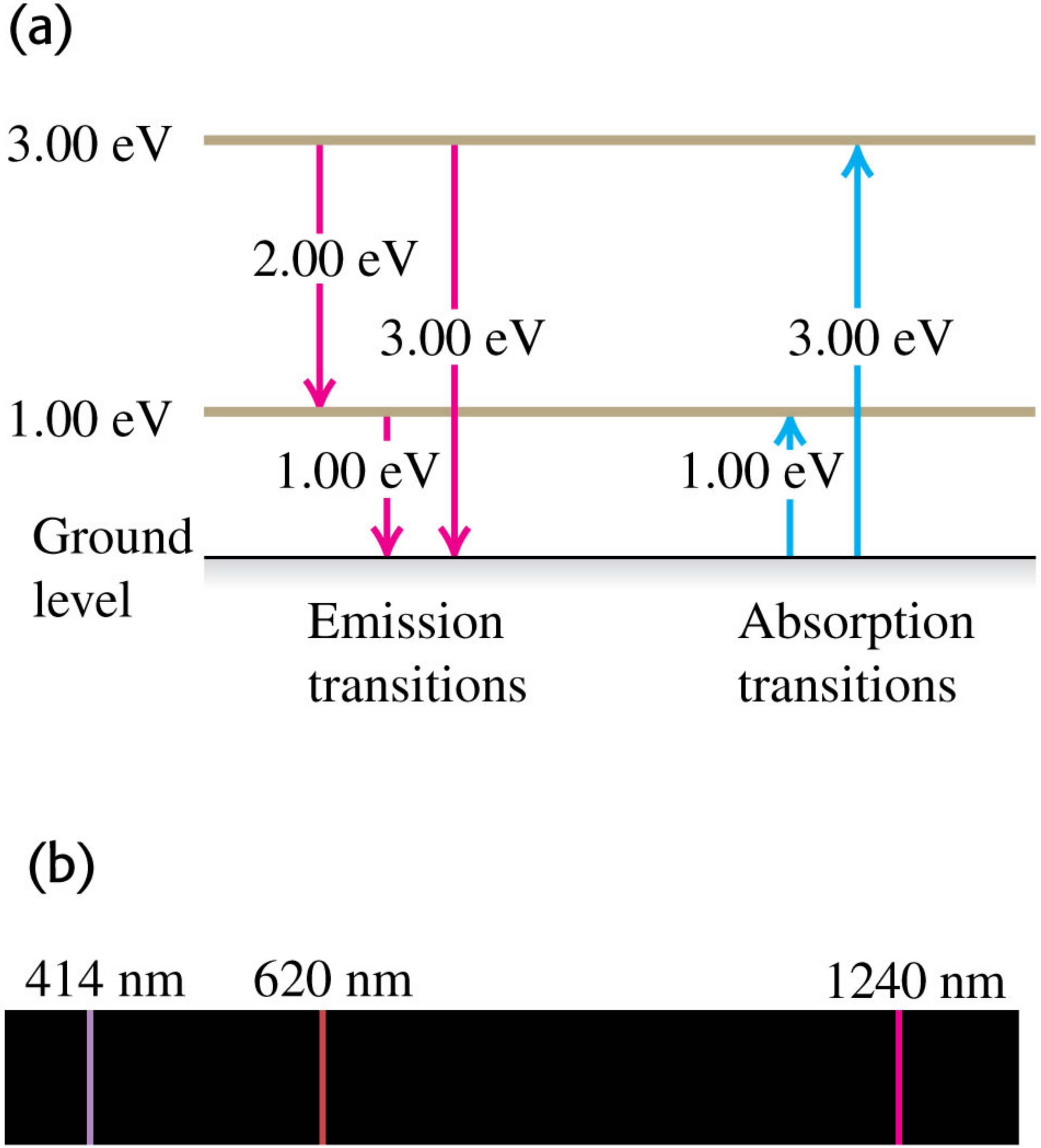

i
E_i
An atom drops from an initial level i to a lower-energy final level f by emitting a photon with energy equal to $E_i - E_f$.
$hf = E_i - E_f$
f
E_f

Figure 39.17



f
E_f
An atom is raised from an initial level i to a higher-energy final level f by absorbing a photon with energy equal to $E_f - E_i$.
$hf = E_f - E_i$
i
E_i

Figure 39.20



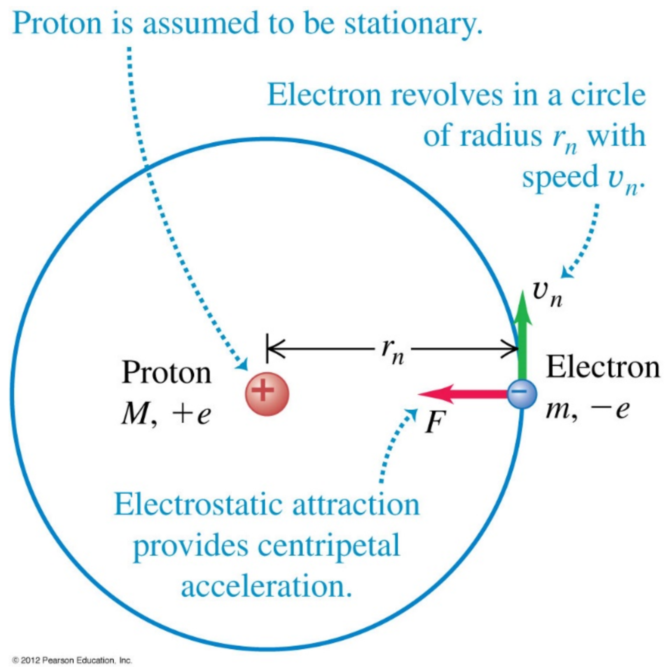
(a)
3.00 eV
2.00 eV
3.00 eV
3.00 eV
1.00 eV
1.00 eV
1.00 eV
Ground level
Emission transitions
Absorption transitions
(b)
414 nm
620 nm
1240 nm

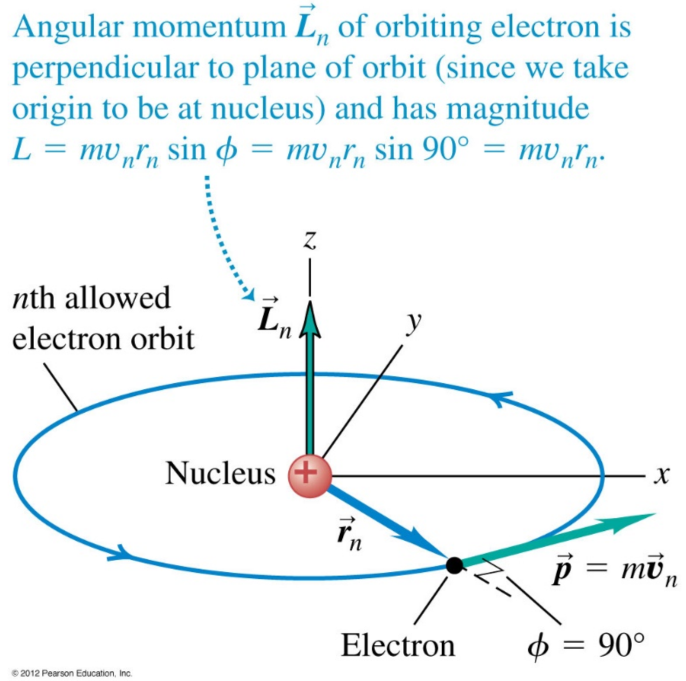
Hydrogen Atom (Bohr):

- Phenomenological; a mixture of old and new physics
- Energy levels correspond to stable circular electronic orbits
- Quantum Hypothesis: stable, circular electronic orbits, are those with a *quantized* angular momentum:

$$L_n = mv_n r_n = n\frac{h}{2\pi} = n\hbar,$$
$$n = 1, 2, 3, \ldots$$

$$\text{hbar,} \qquad \hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\ Js$$
$$\hbar c = 197\ eV\ nm = 197\ MeV \cdot fm$$





Hydrogen Atom (Bohr):

- Circular orbits, coulomb force:

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n^2} = m\frac{v_n^2}{r_n}$$

- Quantization Condition

$$L_n = m\, v_n\, r_n = n\hbar\,, \qquad n = 1, 2, 3, \dots .$$

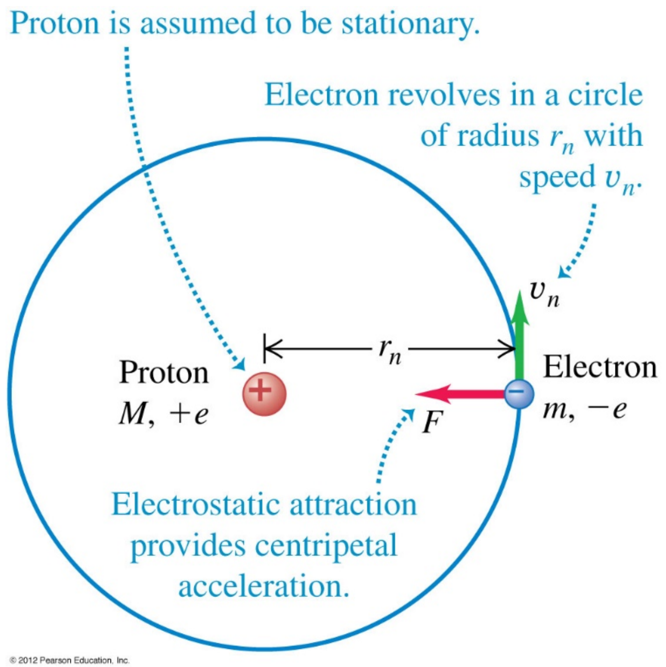
- (…); Results:
  - Stable orbits are labelled by the *quantum number* n

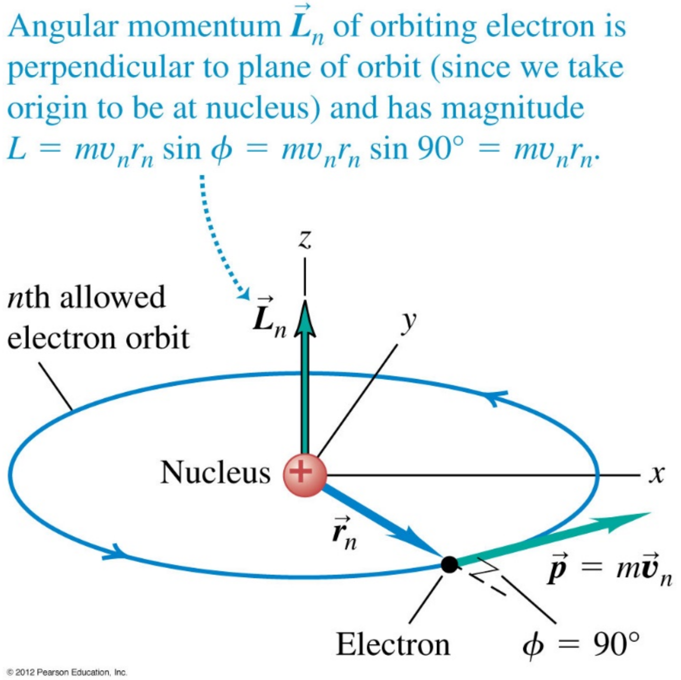
$$r_n = n^2\,\frac{\epsilon_0\, h^2}{\pi\, m\, e^2}, \qquad v_n = \frac{e^2}{2\,\epsilon_0\, h}\frac{1}{n}, \qquad n = 1, 2, \dots$$

  - Ground state, $n = 1$: Bohr Radius

$$a_0 \equiv r_1 = \frac{\epsilon_0\, h^2}{\pi\, m\, e^2} = 5.29 \times 10^{-11}\, m = 0.529\ \text{Å}$$

1 Angstrom (1 Å) = $10^{-10}\ m$





Hydrogen Atom (Bohr):

- Excited states: $r_n = n^2\, a_0$
- Energy levels:
  - Kinetic Energy: $K_n = \frac{1}{2}\, m v_n^2 =$
  - Potential Energy: $U_n = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n} = -\frac{m\, e^4}{4\, h^2}\frac{1}{n^2}$
  - Total Energy:

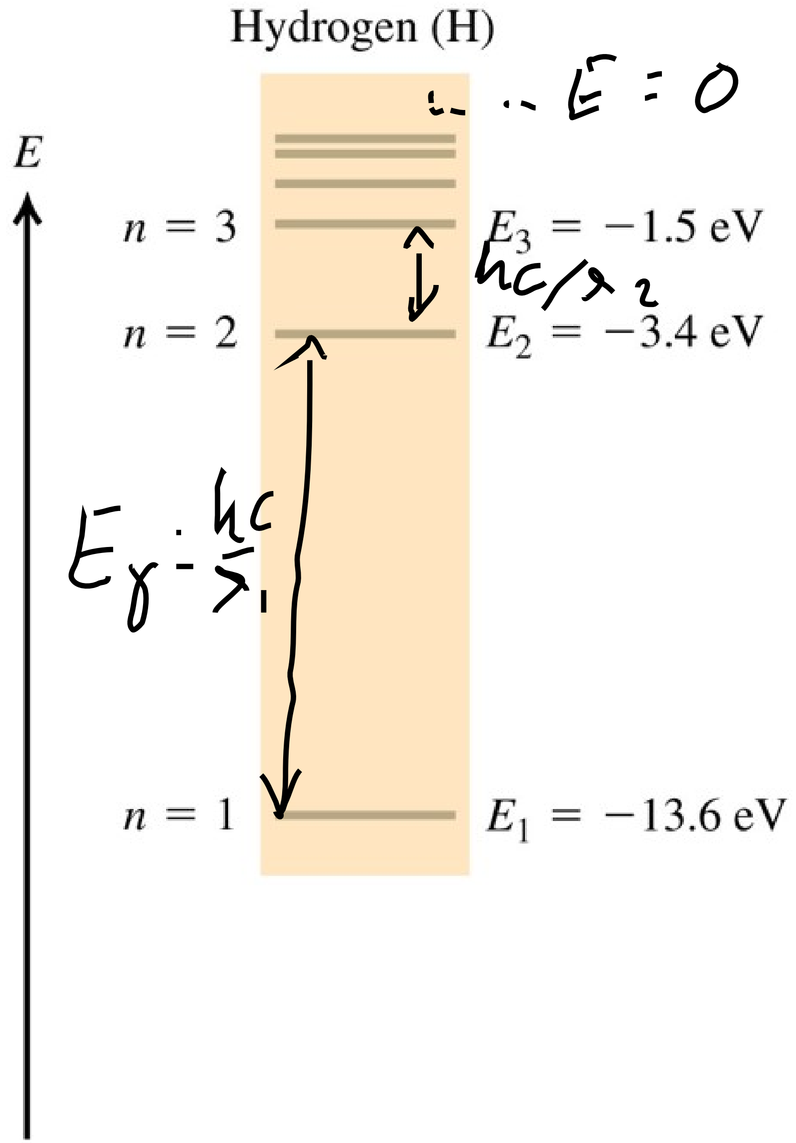
$$E_n = K_n + U_n = -\frac{m\, e^4}{8\epsilon_0^2\, h^2}\frac{1}{n^2}$$

  - Lowest energy level: Ground state

$$n = 1, \qquad E_1 = -\frac{m\, e^4}{8\epsilon_0^2\, h^2} = -13.6\; eV$$

  - Excited states:

$$n = 2, 3, \dots \quad E_n = \frac{E_1}{n^2} = -\frac{13.6\; eV}{n^2}$$



Line Spectrum

- Rydberg constant: $E_1 = -hc\,R$

$$R = \frac{me^4}{8\epsilon_0 h^3 c} = 1.097 \times 10^7\ m^{-1}$$

- Spectral Lines: photon emission/absorption

$$E_\gamma = \frac{hc}{\lambda} = E_{n_U} - E_{n_L} = -hcR\left(\frac{1}{n_U^2} - \frac{1}{n_L^2}\right)$$

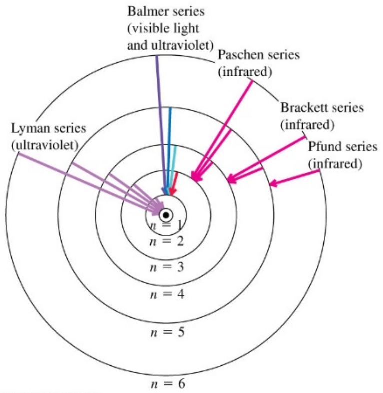
$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_L^2} - \frac{1}{n_U^2}\right)}$$

- Lyman series: Ground state, $n_L = 1 \rightarrow n$

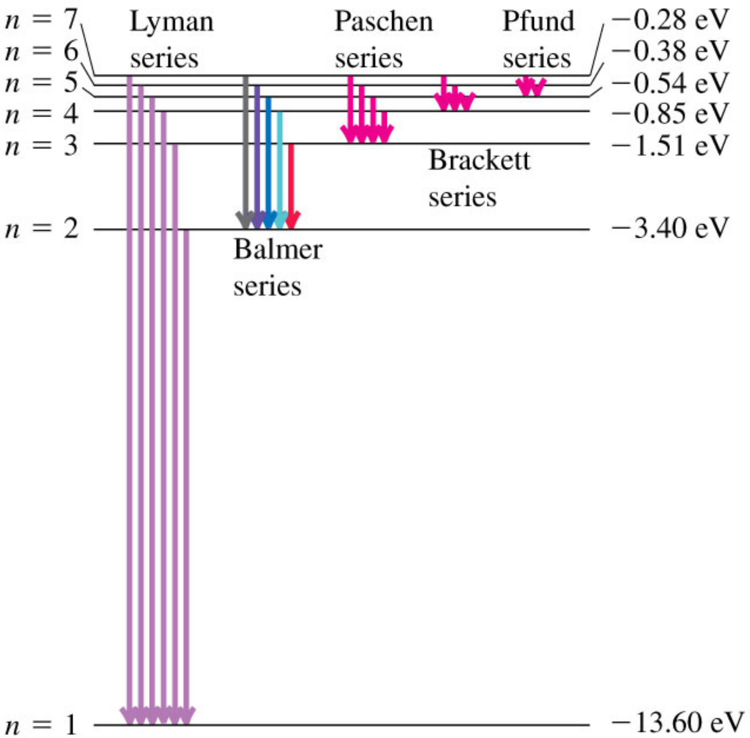
$$\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right)$$

- Ionization Energy (Remove electron from ground state): $n_L = 1 \rightarrow \infty, \quad 13.6\ eV$

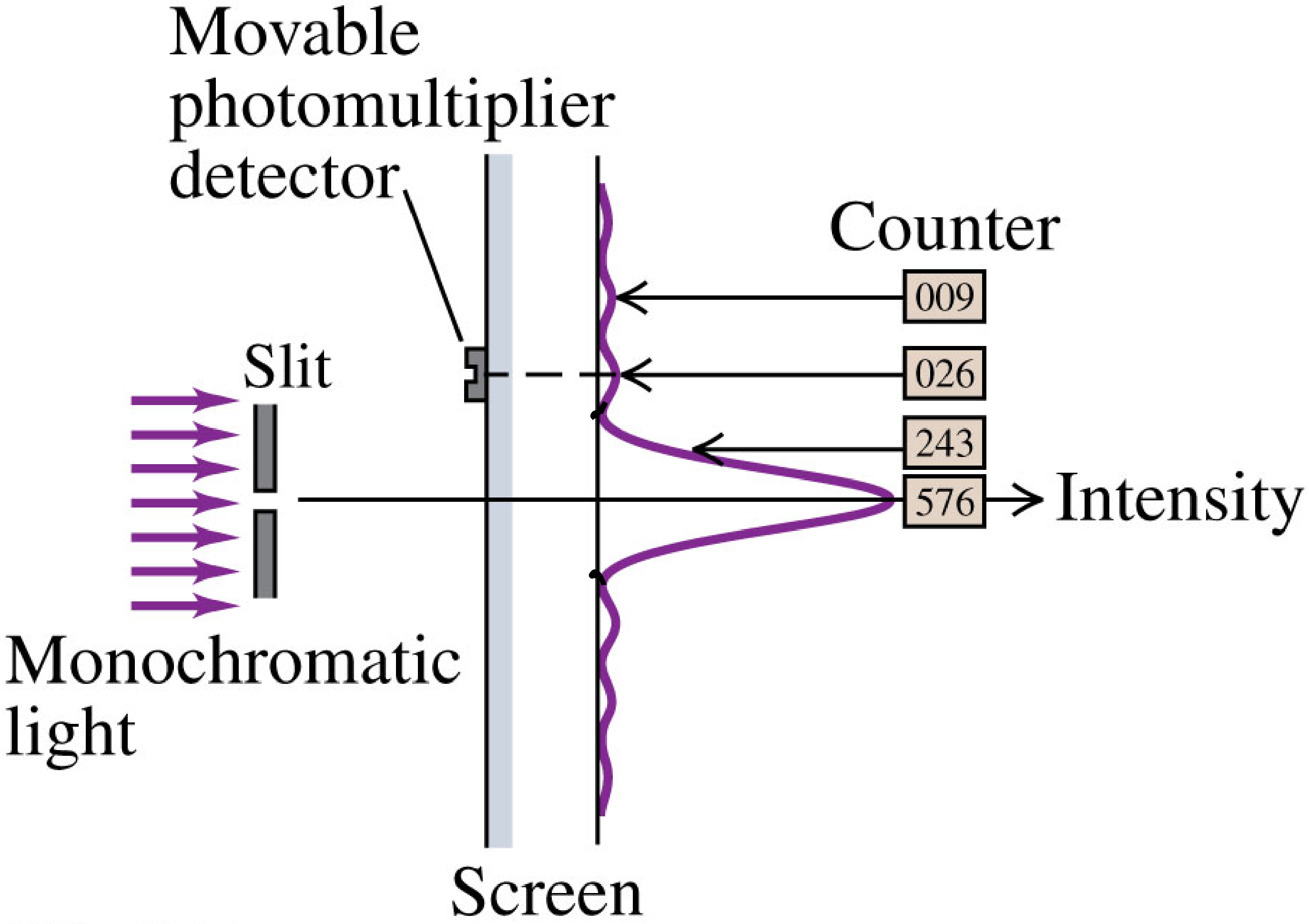
(a) Permitted orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.



(b) Energy-level diagram for hydrogen, showing some transitions corresponding to the various series



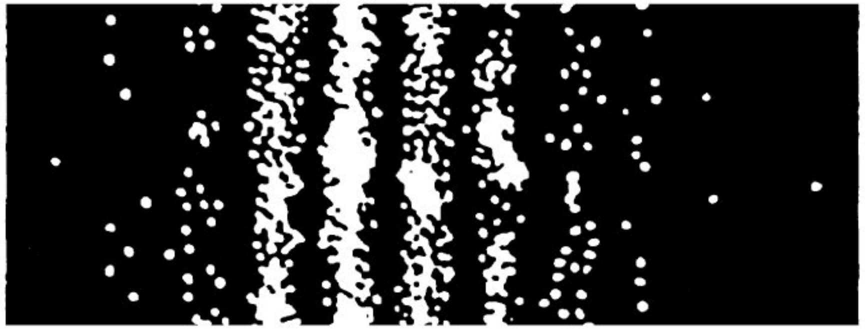
Interference and Diffraction of Light: Photons




Universiteit Leiden
0,025 msec

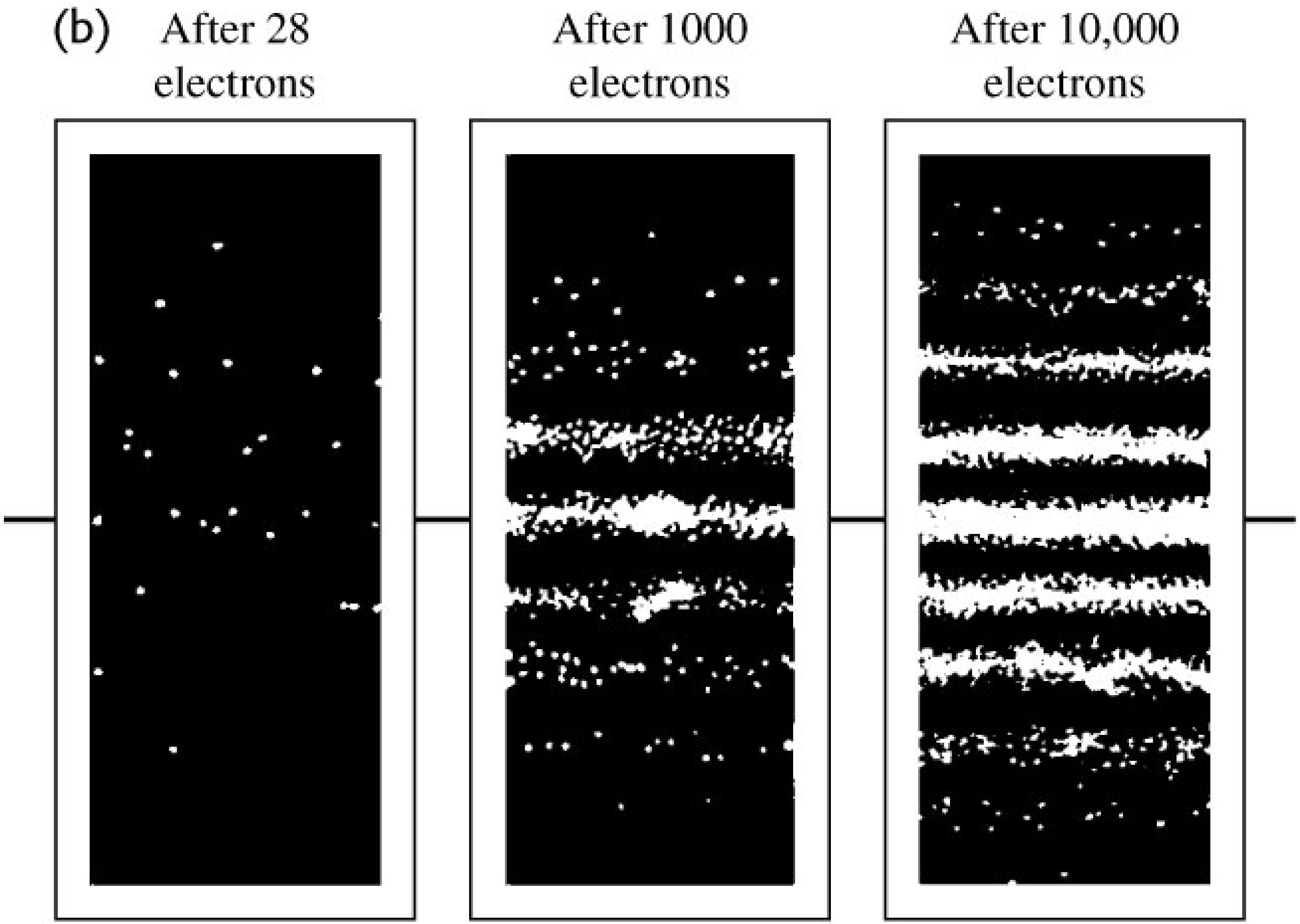
After 21 photons reach the screen

After 1000 photons reach the screen



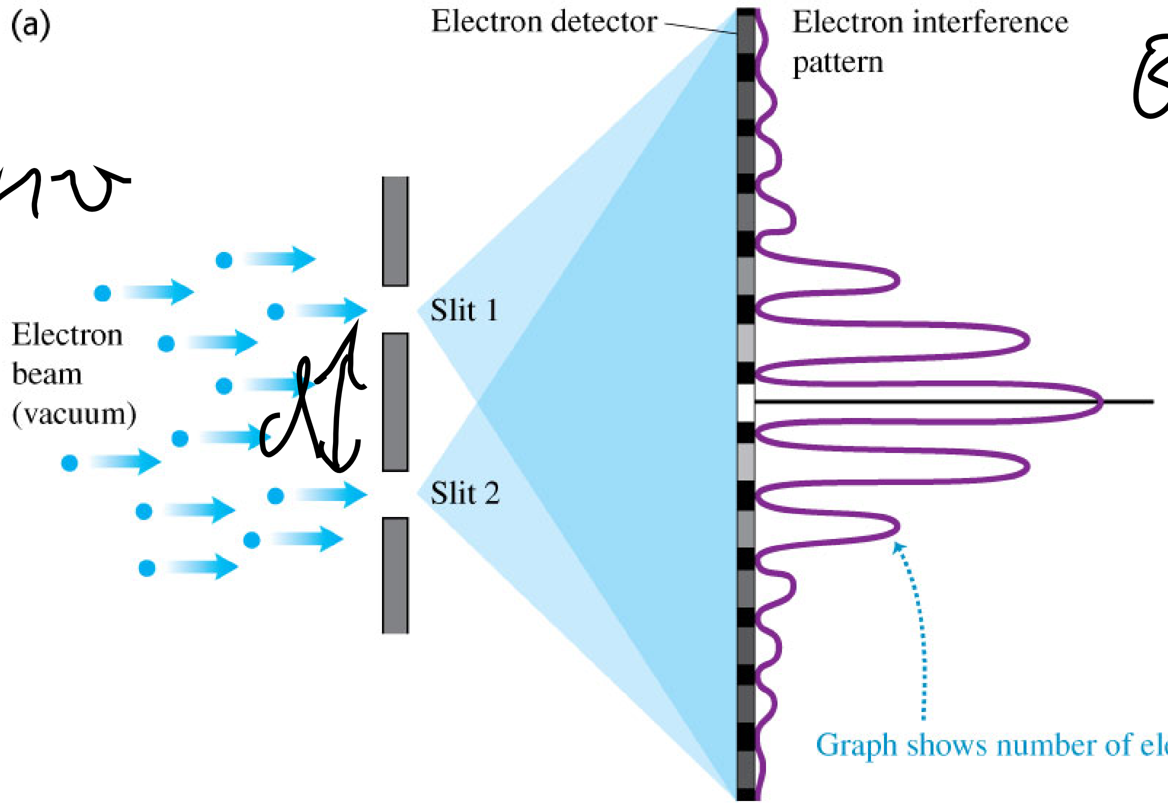
After 10,000 photons reach the screen

Quanta (photons) and waves


(b)
After 28 electrons
After 1000 electrons
After 10,000 electrons

Interference and Diffraction of Matter

$p = mv$

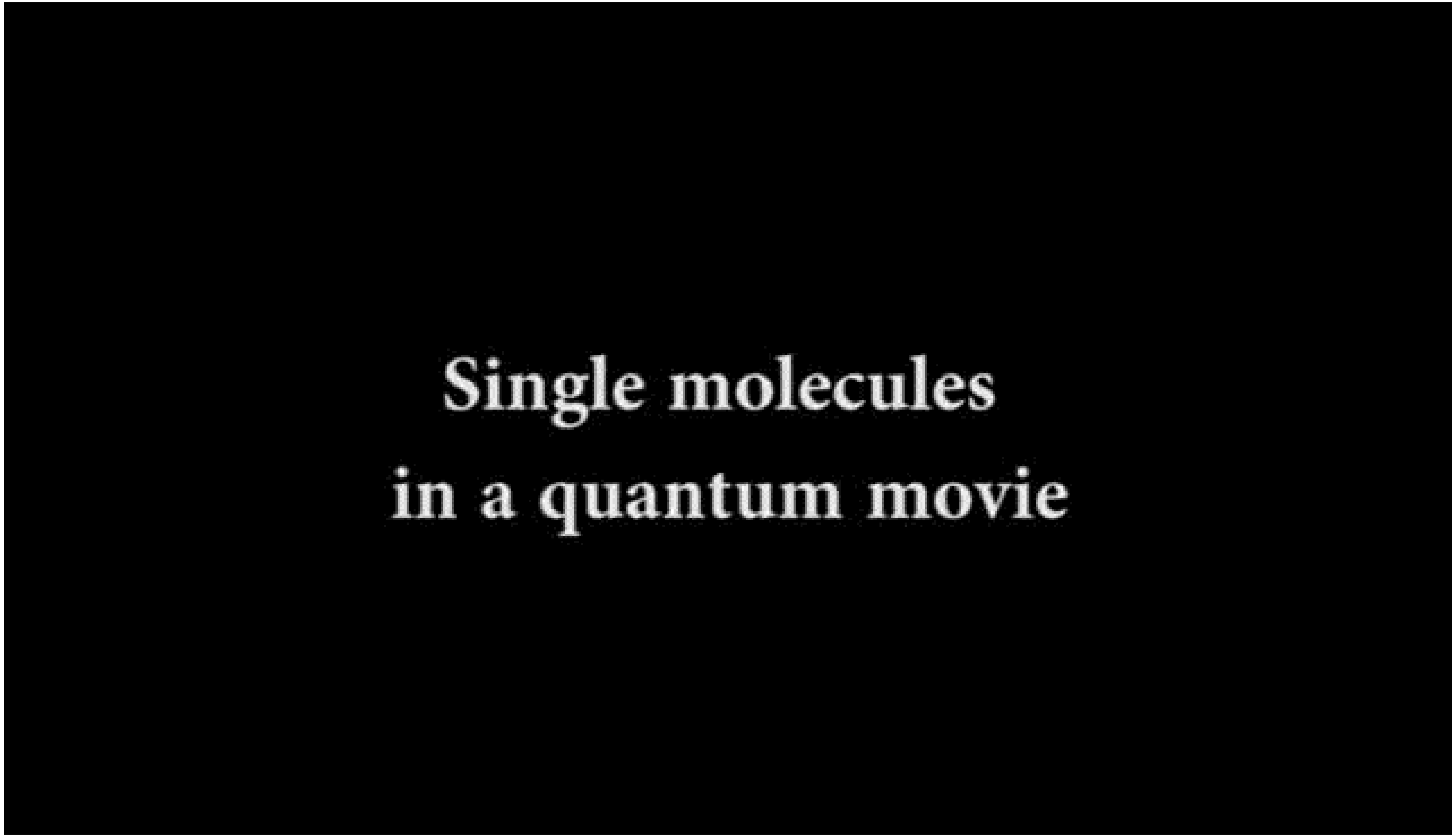


Graph shows number of electrons striking each region of detector.

BRIGHT LINES

$d \sin\theta = m\lambda$

$\lambda = \dfrac{h}{mv}$


Single molecules
in a quantum movie

Matter Waves:

- Matter may display sometimes wave behavior
  - Interference, Diffraction

- De Broglie wavelength (1924):
  - A free particle, mass m, velocity $v \ll c$, will display wave properties with wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Matter Waves: Electron Diffraction

- Example: Electron Diffraction
  - Davisson-Germer (1925)
  - Electrons accelerated by a potential V_{ba} have
    - Energy:

$$eV_{ab} = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

    - Momentum:

$$p = \sqrt{2m(eV_{ba})}$$

    - Wavelength:

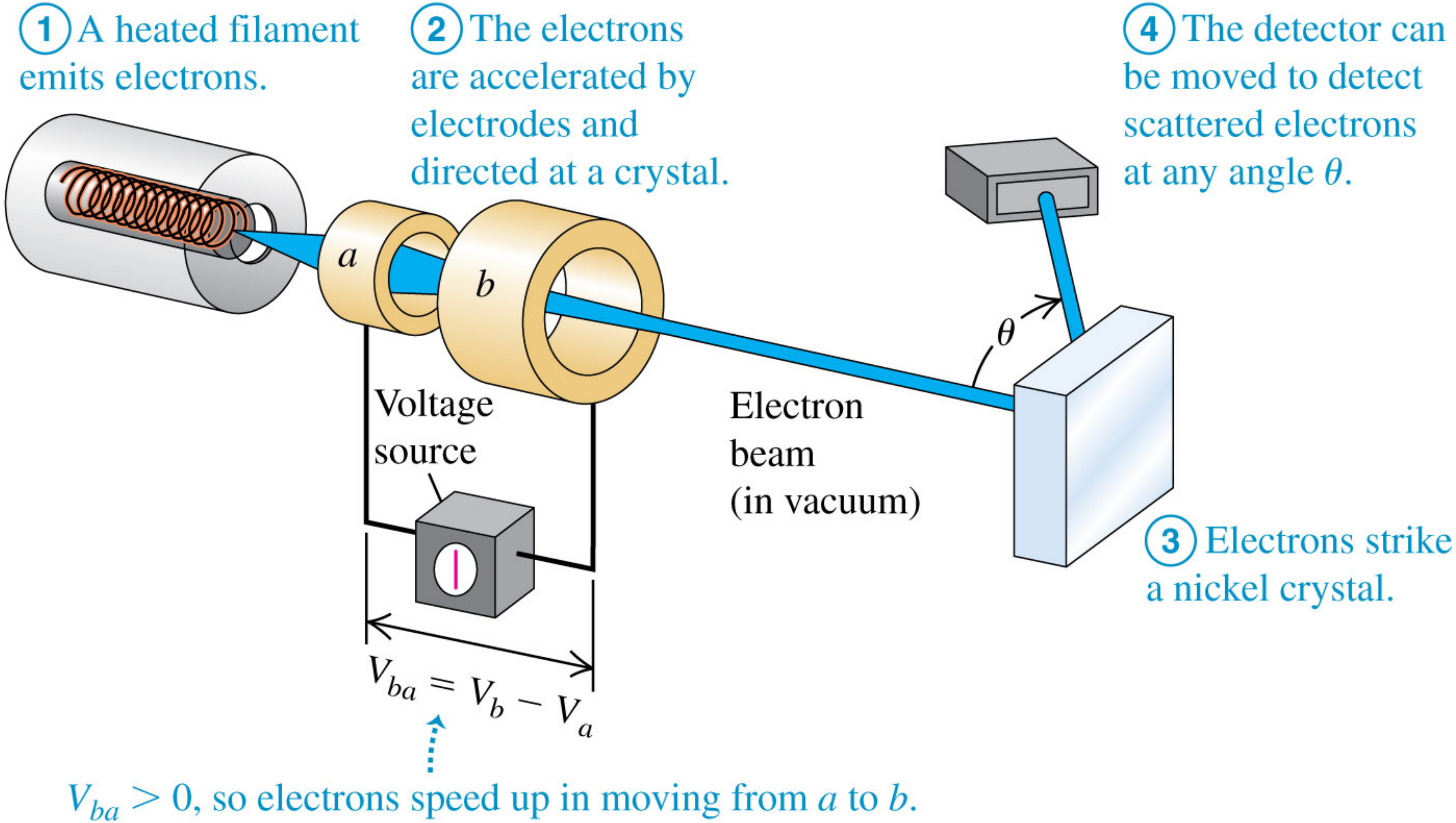
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m(eV_{ba})}}$$

- Diffraction of electrons off the surface of metals with lattice planes separated by d:

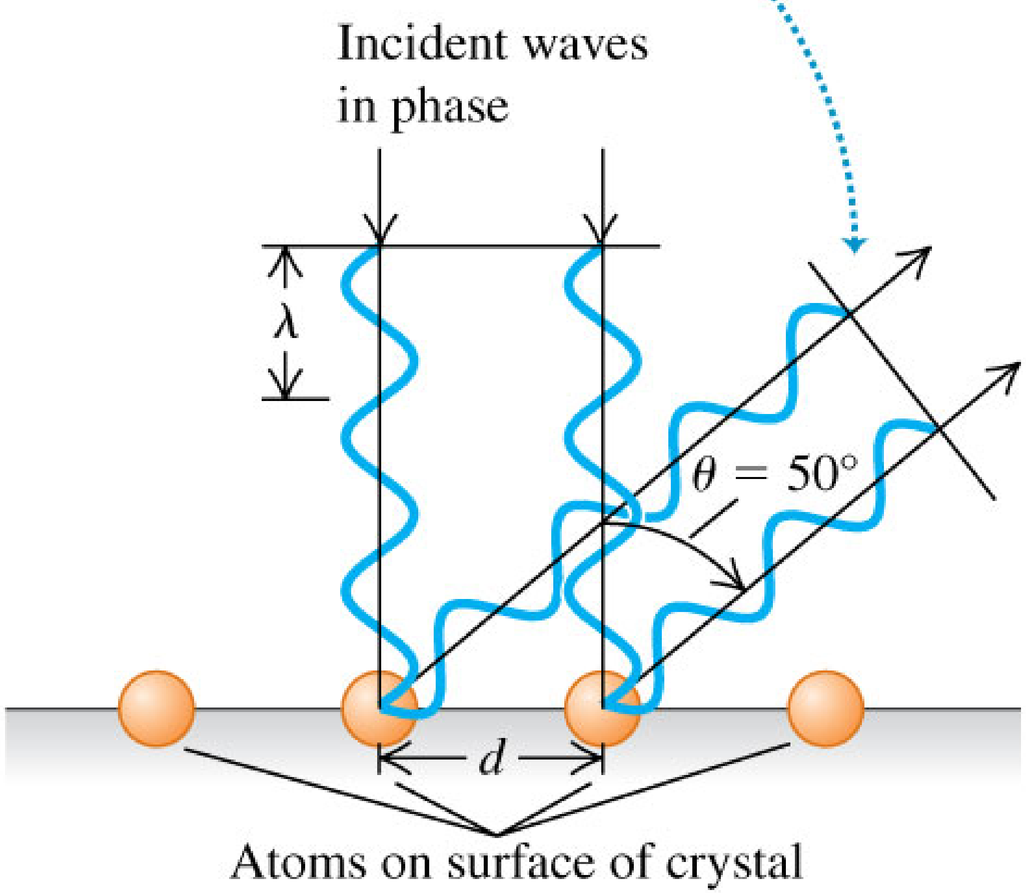
- Electron intensity peaks when

$$d \sin\theta = \overset{n}{\cancel{m}}\,\lambda$$

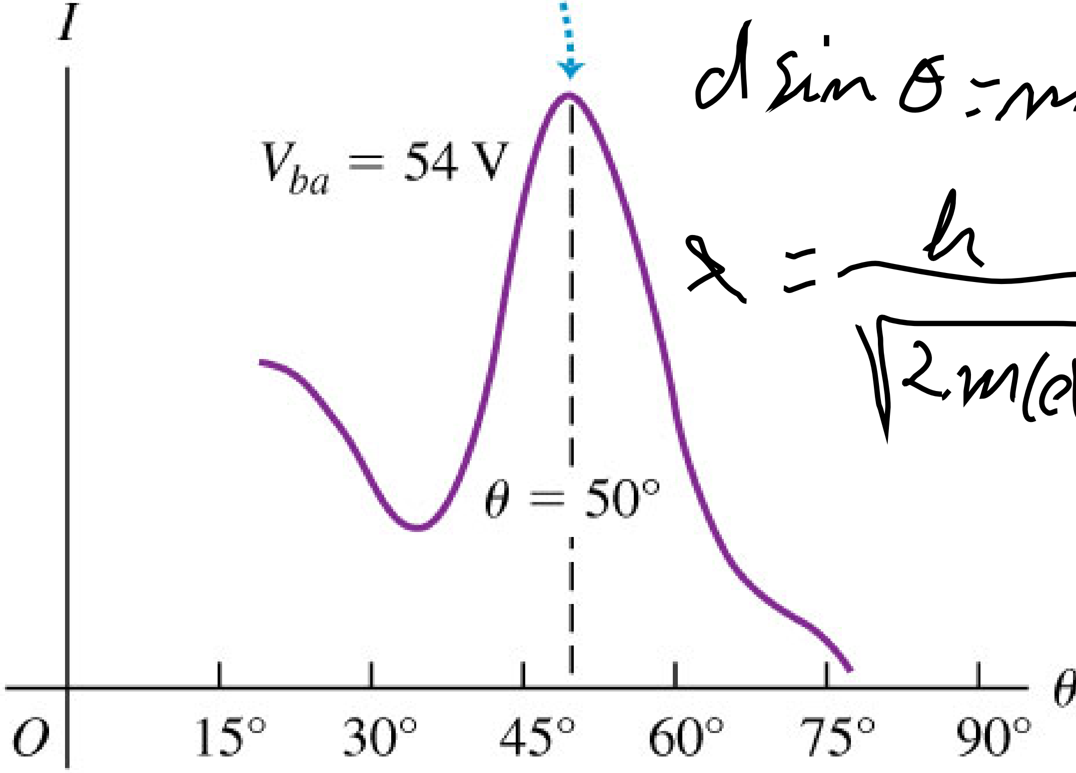
- IT WORKS!


1 A heated filament emits electrons.
2 The electrons are accelerated by electrodes and directed at a crystal.
4 The detector can be moved to detect scattered electrons at any angle θ.
a
b
θ
Voltage source
Electron beam (in vacuum)
3 Electrons strike a nickel crystal.
$V_{ba} = V_b - V_a$
$V_{ba} > 0$, so electrons speed up in moving from a to b.

(b) If the scattered waves are in phase, there is a peak in the intensity of scattered electrons.



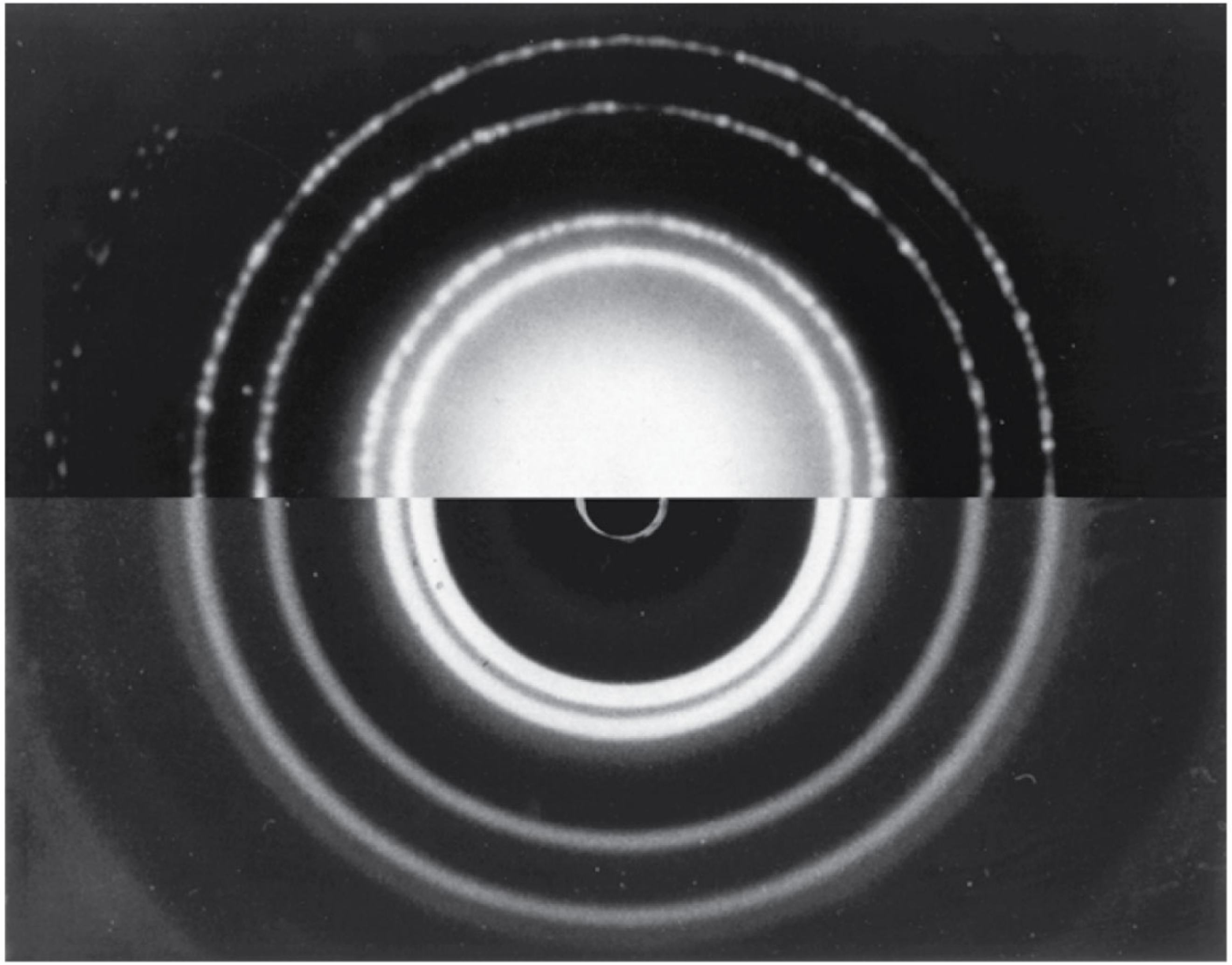
(a) This peak in the intensity of scattered electrons is due to constructive interference between electron waves scattered by different surface atoms.



$$d \sin\theta = m\lambda$$

$$\lambda = \frac{h}{\sqrt{2m(eV_{ba})}}$$

Top: x-ray diffraction



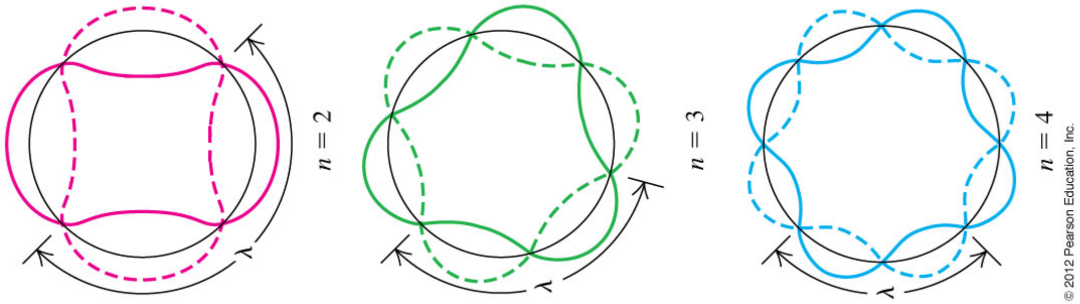
Bottom: electron diffraction

De Broglie Wavelength and Bohr's Hydrogen Atom

- The quantization of the angular momentum in Bohr's model can be readily explained as a consequence of the wave properties of electrons.
- Hydrogen Stable orbits $\leftrightarrow$ Standing waves on a circumference of radius r_n

$$2\pi r_n = n\lambda_n, \qquad n = 1, 2, 3, \ldots$$

$$\lambda_n = \frac{h}{p_n}, \qquad 2\pi r_n = n\,\frac{h}{p_n}$$



$$m v_n r_n = n\frac{h}{2\pi}, \qquad L_n = n\,\hbar, \qquad n = 1, 2, 3, \ldots$$

Heisenberg Uncertainty Principle

- Complementary Principle (Bohr):
  - Systems have complementary properties which cannot be measured accurately at the same time
  - Wave/Particle duality (one or the other but not both at once!):
  - Heisenberg Uncertainty Principle:
    - Some properties are complementary and are not completely well defined in a quantum system
    - Position, velocity uncertainties: $\Delta x, \Delta p$

$$\Delta x \cdot \Delta p \geq \hbar/2$$

    - Energy, time uncertainties: $\Delta E, \Delta \mathrm{t}$

$$\Delta E \cdot \Delta \mathrm{t} \geq \hbar/2$$

    - (…)

Example: Particle in a Box

- Particle confined to be in the region $0 < x < L$
    - Uncertainty in position: $\Delta x \leq L$
- Average momentum, $< p_x > = 0$
    - Uncertainty in momentum: $\Delta p = \sqrt{< p_x^2 > - < p_x >^2} = \sqrt{< p_x^2 >} = p_{rms}$

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \qquad L\,(\Delta p_{min}) \geq \frac{\hbar}{2}, \qquad (\Delta p_{min}) = p_{min} \geq \frac{h}{4\pi L}$$

- A particle confined has a Ground State Energy:

$$E_{min} = \frac{p_{min}^2}{2m} \geq \frac{h^2}{16\pi m L^2}$$